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THE WILCOXON TWO SAMPLE STATISTIC

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ERRATA SHEET

Page	Line	Comments
1	6	Lower case "o" to upper case "O".
1	16	Period to comma and upper case "O" to lower case "o".
1	18	"all real" to "some".
1	25	Omit "the distribution of".
1	27	"parenthesis" to "parentheses".
4	26	$\delta = \alpha$ to $\delta = \gamma$
4	27	$\delta = \gamma$ to $\alpha = \gamma$
5		Substitute Chart I (revised)
6	9	α to .025
6	13	α to .05
7	19	$\begin{matrix} 9 & 0 & +1 \\ & \frac{1}{2} & +1/0 \end{matrix}$ to $\begin{matrix} 9 & 0 & +1 \\ & \frac{1}{2} & +1/0 \end{matrix}$
7	42	Add "(or additional continuity correction)"
8	2	Lower case "t" to upper case "T".
17	1	$\delta = .005$ to $\delta = \gamma$
33	40	Annals of Eugenics to <u>Annals</u> <u>of</u> <u>Eugenics</u>.
34	9	"l;egalite" to "l'egalite".
36	35	7(1952) to 8(1952).
37	4	"Stanford" to "Stamford".
37	5	"Stanford" to "Stamford".

Table II gives values of $\delta = \gamma$, not $\delta = \alpha$.

Table III gives values of $\alpha = \gamma$, not $\delta = \gamma$.

1. Summary.

This paper presents tables for the Wilcoxon two sample statistic and considers the normal approximation to the Wilcoxon two sample statistic. The primary purposes of this paper are a compilation of existing tables of the exact distribution of the Wilcoxon statistic and a comparison of the exact distribution (in the tails) to the normal approximation. Of secondary importance are some additional exact percentage points computed for the Wilcoxon statistic. The remainder of this paper is divided into the following sections: (2) Wilcoxon two sample statistic, (3) Existing tables of the exact distribution of the statistic, (4) Description of tables presented here, (5) Normal approximation, (6) Continuity correction, (7) Computational method, (8) Tables, and (9) Bibliography.

2. Wilcoxon two sample statistic.

Given two independent random samples $X_1,\ldots,X_m$ and $Y_1,\ldots,Y_n$ from two populations with unknown cumlative distribution functions, F and G, respectively, One can test the hypothesis H_0: $F(t) = G(t)$ against the one-sided alternative hypothesis H_A: $F(t) \geq G(t)$ and $F(t) \neq G(t)$ for ~~all real~~ some t. Wilcoxon (55)[1] proposed the statistic $\sum_{i=1}^{m} r_i$ where $r_1,\ldots,r_m$ are the X ranks in the combined sample. Equivalent to the Wilcoxon two sample statistic is U, the Mann-Whitney statistic, the number of pairs (X_i,Y_j) for which $Y_j < X_i$. For the one-sided test the rejection region is $U \leq t$ where $t = t_{\alpha,m,n}$ is the greatest integer for which $P(U \leq t|H_0) \leq \alpha$, the desired level of significance. Under H_0 Mann and Whitney (27) proved that ~~the distribution of~~ U is asymptotically normally distributed when m and n approach infinity. Under H_0 the mean

1. The number in the parathesis refers to the listed references in the bibliography.

and the variance of U are $mn/2$ and $mn(m + n + 1)/12$, respectively.

3. Existing tables of the exact distribution of the statistic.

Chart I shows the extent of existing tables and the extent of the tables presented here.

The present tables are compared with the previous as follows:

a) Mann-Whitney's tables (27) for $\alpha = .005, .01, .025, .05$ over the range $\text{Max.}(m,n) \leq 8$.

b) van der Vaart's tables (46) for $\alpha = .005, .01, .025, .05$ over the range $\text{Max.}(m,n) \leq 10$.

c) Auble's tables (1) for $\alpha = .005, .01, .025, .05$ over the range $\text{Min.}(m,n) \leq 12$ and $\text{Max.}(m,n) \leq 20$.

d) Natrella's tables[2] (29) for $\alpha = .005, .01, .025$ over the range $\text{Min.}(m,n) \leq 12$ and $m + n \leq 30$.

Let t be the largest integer such that $P(U \leq t|H_0) \leq \alpha$. Then the differences with the previous tables are as follows:

for $\alpha = .005$

$m = 7, n = 5$

van der Vaart (46)	$t = 2$
given value	$t = 1$

$m = 16, n = 13$

Natrella (29)	$t = 46$
given value	$t = 45$

$m = 21, n = 5$

Natrella (29)	$t = 14$
given value	$t = 13$

2. Since Natrella's tables encompass White's (52) for the part compared here of the two only Natrella's is checked.

for $\alpha = .01$

$m = 8, n = 7$

Mann-Whitney (27)	$t = 8$[3]
given value	$t = 7$

$m = 8, n = 8$

Mann-Whitney (27)	$t = 10$[4]
given value	$t = 9$

$m = 18, n = 9$

Natrella (29)	$t = 35$
given value	$t = 36$

for $\alpha = .025$

$m = 11, n = 2$

Natrella (29)	$t = 1$
given value	$t = 0$

$m = 13, n = 13$

Natrella (29)	$t = 46$
given value	$t = 45$

$m = 15, n = 15$

Natrella (29)	$t = 65$
given value	$t = 64$

$m = 16, n = 11$

Natrella (29)	$t = 48$
given value	$t = 47$

$m = 17, n = 14$

Auble (1)	$t = 67$
given value	$t = 69$

3. Mann-Whitney gives $P(U \leq 8 | H_0) = .010$ to three places, but to six places is .010256.

4. Mann-Whitney gives $P(U \leq 10 | H_0) = .010$ to three places, but to six places is .010335.

$m = 20,\ n = 4$

Auble (1)	$t = 13$
given value	$t = 14$

$n = 21,\ n = 9$

Natrella (29)	$t = 50$
given value	$t = 51$

for $\alpha = .05$

$m = 9,\ n = 3$

Auble (1)	$t = 3$
given value	$t = 4$

$n = 19,\ n = 7$

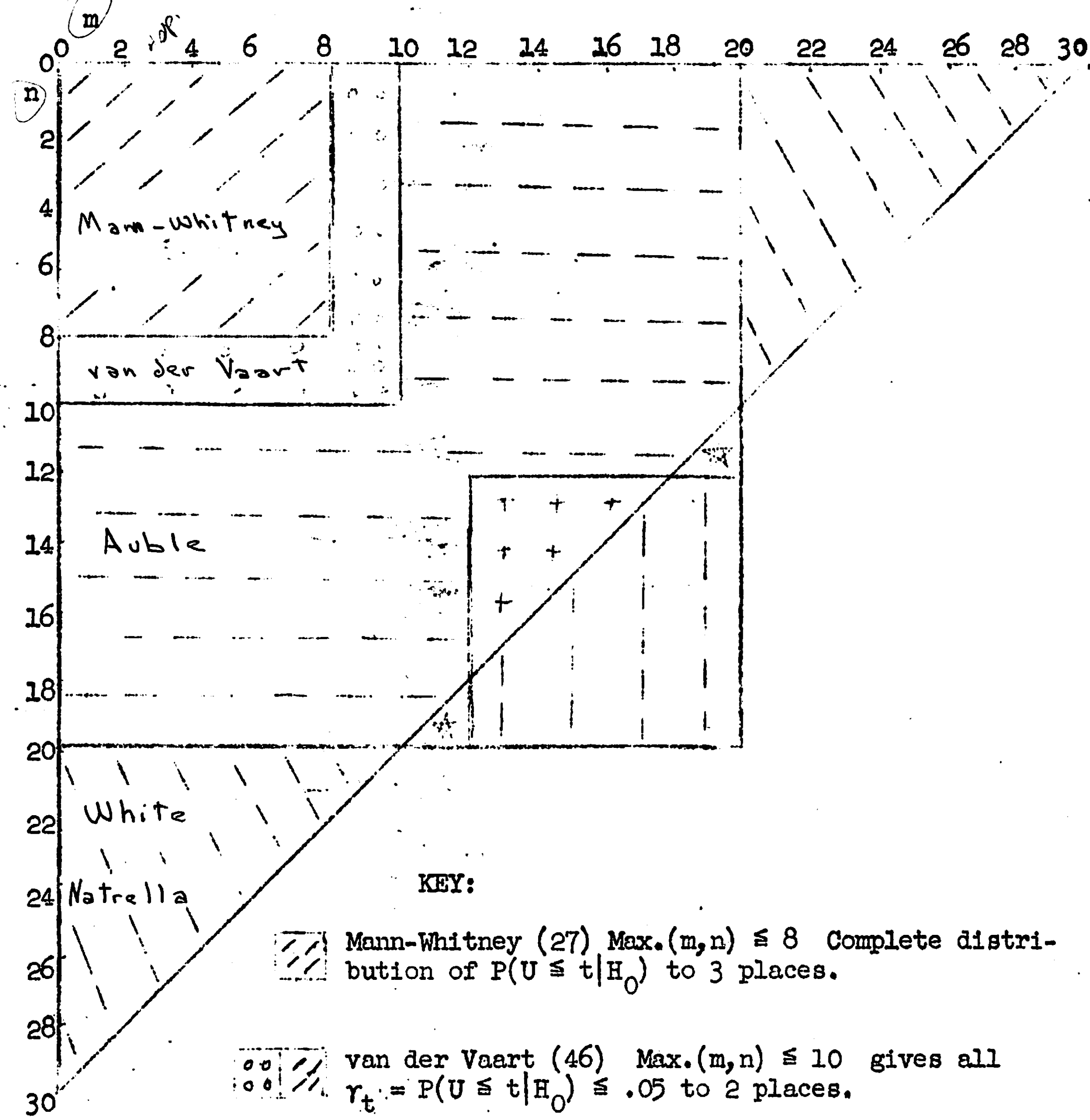
Auble (1)	$t = 27$[5]
given value	$t = 37$

4. Description of tables presented here.

Working under the null hypothesis, H_0, and considering only the lower tail of the distribution, the significant values of the Wilcoxon two sample test and the normal approximation to the Wilcoxon two sample probability distribution have been computed and compared for the levels of significance of 0.5, 1, 2.5, and 5 per cent, and those m and n for which $\mathrm{Min.}(m,n) \leq 12$ and $m + n \leq 30$. Table I shows the largest integer, t, for which $\gamma = P(U \leq t|H_0) \leq \alpha$, the desired level of significance, and the corresponding normal approximation. (The t's corresponding to the exact values outside the range of my computation - $\mathrm{Min.}(m,n) \leq 12$ and $m + n \leq 30$ - are taken from Auble (1).) Table II shows $\delta - \gamma$ where $\delta = P(U \leq t + 1|H_0) > \alpha$. Table III shows $\alpha - \gamma$.

5. Typographical error.

Chart I
Existing tables of the Wilcoxon two sample statistic and levels of significance for the one-sided test.



KEY:

Mann-Whitney (27) Max.$(m,n) \leq 8$ Complete distribution of $P(U \leq t|H_0)$ to 3 places.

van der Vaart (46) Max.$(m,n) \leq 10$ gives all $\gamma_t = P(U \leq t|H_0) \leq .05$ to 2 places.

Auble (1) Max.$(m,n) \leq 20$ gives the largest integer t for $P(U \leq t|H_0) \leq \alpha$ where α = .001, .005, .01, .02, .025, .04, .05, .10.

White (52) $m + n \leq 30$ gives the largest integer t for $P(U \leq t|H_0) \leq \alpha$ where α = .0005, .005, .025.

Natrella (29) $m + n \leq 30$ gives the largest integer t for $P(U \leq t|H_0) \leq \alpha$ where α = .005, .01, .025.

Jacobson Min.$(m,n) \leq 12$ and $m + n \leq 30$ gives the largest integer t for $P(U \leq t|H_0) \leq \alpha$ where α = .005, .01, .025, .05.

5. Normal approximation.

The notation and restrictions of section 4 will be used here. At the .005 level of significance the normal approximation is t or t - 1 for the minimum of sample sizes greater than 3. At the .01 level of significance the normal approximation is t (most often) or t + 1 for the minimum of the two sample sizes greater than 3. At the .025 level of significance the normal approximation is t and t + 1 (most often). Hence for the sample sizes considered using the normal approximation minus one will yield a level of significance less than or equal to ~~α~~ .025. At the .05 level of significance the normal approximation is t + 1 or t + 2 (most often). Hence for the sample sizes considered using the normal approximation minus two will yield a level of significance less than or equal to ~~α~~ .05.

6. Continuity correction.

Among the possible combinations of continuity correction and rounding rule obtained by using a continuity correction, x, which can be represented as a fraction with denominator 2 and using a rounding rule of round up, round down, or round according to the usual rule there are two distinct sets with respect to the differences between the normal approximation and the exact values for the Wilcoxon two sample test. A continuity correction of x and round down, $x + \frac{1}{2}$ and round according to the usual rule, or x + 1 and round up give the same result. Substituting x - 1 or x + 1 for x in the previous statement will yield one more or one less, respectively, than the previous result. Changing x by one half plus an integer yields the second set. Table A shows the number of differences between the normal approximation and the exact value of the Wilcoxon two sample test for continuity corrections

Table A

The Number of Differences Between the Exact Values of the Wilcoxon Test and the Normal Approximation for $1 \leq n \leq m \leq 20$.

		levels of significance							
	continuity	.05		.025		.01		.005	
m	correction	C.F.[a]	D/T[b]	C.F.	D/T	C.F.	D/T	C.F.	D/T
3	0	+1	0/1	-	-	-	-	-	-
	½	0	0/1	-	-	-	-	-	-
4	0	+1	0/2	+1	0/1	-	-	-	-
	½	+1	0/2	+1	0/1	-	-	-	-
5	0	+1	0/4	+1	0/3	+1/0[c]	1/2	0	0/1
	½	0	1/4	0	1/3	0	0/2	0	0/1
6	0	+1	0/5	+1	1/4	0	0/3	0	0/3
	½	0	2/5	0	1/4	0	0/3	+1	0/3
7	0	+1	0/6	+1	1/5	0	2/5	0	1/4
	½	+1	1/6	0	2/5	0	0/5	0	1/4
8	0	+1	0/7	0	3/7	+1/0	3/6	0	0/5
	½	0	3/7	0	1/7	0	1/6	0	2/5
9	0	+1	0/8	+1	1/8	0	2/7	0	0/7
	½	+1/0[c]	4/8	0	2/8	0	2/7	0	1/7
10	0	+1	0/9	+1	1/9	+1/0	4/8	0	0/8
	½	+1	3/9	0	2/9	0	0/8	-1	3/8
11	0	+1	1/10	+1	1/10	+1	4/9	0	1/9
	½	+1/0	5/10	0	4/10	0	2/9	-1	4/9
12	0	+1	3/11	+1	4/11	0	3/10	0	2/10
	½	+1	3/11	+1	4/11	0	3/10	-1	2/10
13	0	+1	0/12	+1	2/12	0	2/12	0	4/11
	½	+1	3/12	0	2/12	0	3/12	-1	3/11
14	0	+1	2/13	+1	2/13	0	6/13	0	1/12
	½	+1	4/13	0	3/13	0	3/13	-1	4/12
15	0	+1	3/14	+1	2/14	0	3/14	0	3/13
	½	0	6/14	0	3/14	0	5/14	-1	4/13
16	0	+1	0/15	+1	6/15	0	1/15	0	3/14
	½	+1	6/15	0	3/15	0	7/15	-1	5/14
17	0	+1	4/16	+1	3/16	0	2/16	0	5/15
	½	0	7/16	0	3/16	0/-1	8/16	-1	1/15
18	0	+1	2/17	+1	4/17	0	4/17	0	6/16
	½	+1	7/17	0	4/17	0	7/17	-1	2/16
19	0	+1	1/19	+1	4/18	0	2/18	-1	8/18
	½	+1	6/19	0	4/18	0	5/18	-1	8/18
20	0	+1	1/20	+1	3/19	0	5/19	-1	9/19
	½	0	9/20	0	3/19	0	8/19	-1	5/19

a. correction factor, b. difference/total, c. +1 or 0.
(or additional continuity correction)

of 0 and $\frac{1}{2}$ and round according to the usual rule and for $1 \leq n \leq m \leq 20$. Checking Table A a continuity correction of 1 and round according the usual rule is best for levels of significance .05 and .025 and a continuity correction of 0 and round according to the usual rule is best for levels of significance .01 and .005.

7. Computational method.

The exact probabilities for the Wilcoxon two sample test were computed using the method - a difference equation in three variables and two auxilary tables - shown by Fix and Hodges (11). Computations were continued as far as practicable with a desk computor which is dependent on the extent of the auxilary tables given by Fix and Hodges - i.e., Min.$(m,n) \leq 12$. For a variety of methods that may be used to compute the exact probabilities see van der Vaart (48). In table I the normal approximation was computed using the property that $\frac{U - \frac{1}{2}mn}{\sqrt{mn(m + n + 1)/12}}$ approaches $N(0,1)$ as m and n approach infinity. No continuity correction was used and rounding was according to the usual rule.

Table I ($\alpha = .005$)

Exact values for the .005 level of significance and the corresponding normal approximation for Max.(m,n) $\leq$ 20 or m + n $\leq$ 30.

m-n	n = 1		2		3		4		5		6		7		8	
	E[a]	N[b]	E	N	E	N	E	N	E	N	E	N	E	N	E	N
0	-	-	-	-	-	-	-	-	0	0	2	2	4	4	7	7
1	-	-	-	-	-	-	-	-	1	1	3	3	6	6	9	9
2	-	-	-	-	-	-	0	0	1	2	4	4	7	7	11	11
3	-	-	-	-	-	-	0	0	2	2	5	5	9	9	13	13
4	-	-	-	-	-	-	1	1	3	3	6	6	10	10	15	15
5	-	-	-	-	-	-	1	1	4	4	7	7	12	12	17	16
6	-	-	-	-	0	0	2	2	5	5	9	8	13	13	18	18
7	-	-	-	-	0	0	2	2	6	6	10	10	15	14	20	20
8	-	-	-	-	0	0	3	3	7	6	11	11	16	16	22	22
9	-	-	-	-	1	0	3	3	7	7	12	12	18	17	24	24
10	-	-	-	-	1	0	4	4	8	8	13	13	19	19	26	26
11	-	-	-	-	1	1	5	4	9	9	15	14	21	20	28	27
12	-	-	-	-	2	1	5	5	10	10	16	15	22	22	30	29
13	-	-	-	-	2	1	6	5	11	10	17	17	24	23	32	31
14	-	-	-	-	2	1	6	6	12	11	18	18	25	25	34	33
15	-	-	-	-	2	1	7	6	13	12	19	19	27	26		
16	-	-	-	-	3	2	8	7	13	13	21	20	29	28		
17	-	-	0	-3	3	2	8	7	14	14	22	21				
18	-	-	0	-3	3	2	9	8	15	15	23	22				
19	-	-	0	-3	4	2	9	8	16	15						
20	-	-	0	-3	4	2	10	9	17	16						
21	-	-	0	-3	4	3	10	9								
22	-	-	0	-3	5	3	11	10								
23	-	-	0	-3	5	3										
24	-	-	0	-3	5	3										
25	-	-	1	-3												
26	-	-	1	-3												

a. exact value b. normal approximation

Table I (α = .005)

m-n	9 E	9 N	10 E	10 N	11 E	11 N	12 E	12 N	13 E	13 N	14 E	14 N	15 E	15 N	16 E	16 N
0	11	11	16	16	21	20	27	27	34	34	42	42	51	50	60	60
1	13	13	18	18	24	24	31	31	38	38	46	46	55	55	65	64
2	16	16	21	21	27	27	34	34	42	42	50	50	60	59	70	69
3	18	18	24	23	30	30	37	37	45	45	54	54	64	64	74	74
4	20	20	26	26	33	33	41	41	49	49	58	58	68	68	79	79
5	22	22	29	29	36	36	44	44	53	53	63	62	73	73		
6	24	24	31	31	39	39	47	47	56	56	67	66				
7	27	26	34	34	42	42	51	50	60	60						
8	29	29	37	36	45	45	54	54								
9	31	31	39	39	48	48										
10	33	33	42	41												
11	36	35														
12	38	38														

m-n	17 E	17 N	18 E	18 N	19 E	19 N	20 E	20 N
0	70	70	81	81	93	92	105	105
1	75	75	87	86	99	98		
2	81	80	92	92				
3	86	85						

n	1	2	3	4
limit value for normal approximation as $m \to \infty$	-0.244	-0.052	0.212	0.513

Table I (α = .01)

Exact values for the .01 level of significance and the corresponding normal approximation for Max.(m,n) $\leq$ 20 or m + n $\leq$ 30.

m-n	n = 1 E	1 N	2 E	2 N	3 E	3 N	4 E	4 N	5 E	5 N	6 E	6 N	7 E	7 N	8 E	8 N
0	-	-	-	-	-	-	-	-	1	1	3	3	6	6	9	10
1	-	-	-	-	-	-	0	1	2	2	4	5	7	8	11	12
2	-	-	-	-	-	-	1	1	3	3	6	6	9	10	13	14
3	-	-	-	-	-	-	1	2	4	4	7	7	11	11	15	16
4	-	-	-	-	0	0	2	2	5	5	8	9	12	13	17	18
5	-	-	-	-	0	1	3	3	6	6	9	10	14	14	20	20
6	-	-	-	-	1	1	3	4	7	7	11	11	16	16	22	22
7	-	-	-	-	1	1	4	4	8	8	12	12	17	18	24	24
8	-	-	-	-	1	2	5	5	9	9	13	14	19	19	26	26
9	-	-	-	-	2	2	5	5	10	10	15	15	21	21	28	28
10	-	-	-	-	2	2	6	6	11	11	16	16	23	23	30	30
11	-	-	0	-1	2	3	7	7	12	12	18	18	24	25	32	32
12	-	-	0	-1	3	3	7	7	13	13	19	19	26	26	34	34
13	-	-	0	-1	3	3	8	8	14	14	20	20	28	28	36	36
14	-	-	0	-1	4	4	9	9	15	15	22	22	30	30	38	38
15	-	-	0	-1	4	4	9	9	16	16	23	23	31	31		
16	-	-	0	0	4	4	10	10	17	17	24	24	33	33		
17	-	-	1	0	5	5	11	11	18	18	26	26				
18	-	-	1	0	5	5	11	11	19	19	27	27				
19	-	-	1	0	6	5	12	12	20	20						
20	-	-	1	0	6	6	13	13	21	21						
21	-	-	1	0	6	6	13	13								
22	-	-	1	0	7	6	14	14								
23	-	-	1	0	7	7										
24	-	-	1	0	7	7										
25	-	-	2	0												
26	-	-	2	0												

Table I (α = .01)

m-n	9 E	9 N	10 E	10 N	11 E	11 N	12 E	12 N	13 E	13 N	14 E	14 N	15 E	15 N	16 E	16 N
0	14	14	19	19	25	25	31	32	39	39	47	47	56	56	66	66
1	16	17	22	22	28	28	35	35	43	43	51	52	61	61	71	71
2	18	19	24	25	31	31	38	39	47	47	56	56	66	66	76	77
3	21	21	27	27	34	35	42	42	51	51	60	60	70	71	82	82
4	23	24	30	30	37	38	46	46	55	55	65	65	75	75	87	87
5	26	26	33	33	41	41	49	49	59	59	69	69	80	80		
6	28	28	36	36	44	44	53	53	63	63	73	74				
7	31	31	38	39	47	47	56	57	67	67						
8	33	33	41	41	50	50	60	60								
9	36	36	44	44	53	54										
10	38	38	47	47												
11	40	41														
12	43	43														

m-n	17 E	17 N	18 E	18 N	19 E	19 N	20 E	20 N
0	77	77	88	88	101	101	114	114
1	82	83	94	94	107	107		
2	88	88	100	100				
3	93	94						

n	1	2	3	4
limit value for normal approximation as $m \to \infty$	-0.171	0.050	0.337	0.657

Table I ($\alpha = .025$)

Exact values for the .025 level of significance and the corresponding normal approximation for Max.$(m,n) \leq 20$ or $m + n \leq 30$.

m-n	n: 1		2		3		4		5		6		7		8	
	E	N	E	N	E	N	E	N	E	N	E	N	E	N	E	N
0	-	-	-	-	-	-	0	1	2	3	5	6	8	9	13	13
1	-	-	-	-	-	-	1	2	3	4	6	7	10	11	15	16
2	-	-	-	-	0	1	2	3	5	5	8	9	12	13	17	18
3	-	-	-	-	1	1	3	4	6	7	10	10	14	15	19	20
4	-	-	-	-	1	2	4	4	7	8	11	12	16	17	22	23
5	-	-	-	-	2	2	4	5	8	9	13	13	18	19	24	25
6	-	-	0	0	2	3	5	6	9	10	14	15	20	21	26	27
7	-	-	0	1	3	3	6	7	11	11	16	17	22	23	29	30
8	-	-	0	1	3	4	7	8	12	13	17	18	24	24	31	32
9	-	-	0	1	4	4	8	9	13	14	19	20	26	27	34	34
10	-	-	1	1	4	5	9	10	14	15	21	21	28	29	36	37
11	-	-	1	1	5	5	10	10	15	16	22	23	30	31	38	39
12	-	-	1	2	5	6	11	11	17	17	24	25	32	33	41	41
13	-	-	1	2	6	6	11	12	18	19	25	26	34	35	43	44
14	-	-	1	2	6	7	12	13	19	20	27	28	36	37	45	46
15	-	-	2	2	7	7	13	14	20	21	29	29	38	39		
16	-	-	2	2	7	8	14	15	22	22	30	31	40	41		
17	-	-	2	3	8	9	15	16	23	24	32	33				
18	-	-	2	3	8	9	16	16	24	25	33	34				
19	-	-	3	3	9	10	17	17	25	26						
20	-	-	3	3	9	10	17	18	27	27						
21	-	-	3	3	10	11	18	19								
22	-	-	3	4	10	11	19	20								
23	-	-	3	4	11	12										
24	-	-	4	4	11	12										
25	-	-	4	4												
26	-	-	4	4												

Table I ($\alpha = .025$)

m-n	n = 9 E	9 N	10 E	10 N	11 E	11 N	12 E	12 N	13 E	13 N	14 E	14 N	15 E	15 N	16 E	16 N
0	17	18	23	24	30	31	37	38	45	46	55	55	64	65	75	76
1	20	21	26	27	33	34	41	42	50	51	59	60	70	70	81	82
2	23	24	29	30	37	38	45	46	54	55	64	65	75	76	86	87
3	26	26	33	33	40	41	49	50	59	59	69	70	80	81	92	93
4	28	29	36	37	44	45	53	54	63	64	74	74	85	86	98	98
5	31	32	39	40	47	48	57	58	67	68	78	79	90	91		
6	34	35	42	43	51	52	61	62	72	72	83	84				
7	37	37	45	46	55	55	65	66	76	77						
8	39	40	48	49	58	59	69	70								
9	42	43	52	52	62	63										
10	45	46	55	55												
11	48	48														
12	51	51														

m-n	n = 17 E	17 N	18 E	18 N	19 E	19 N	20 E	20 N
0	87	88	99	100	113	113	127	128
1	93	94	106	106	119	120		
2	99	100	112	113				
3	105	106						

	n = 1	2	3	4
limit value for normal approximation as $m \to \infty$	-0.066	0.200	0.520	0.868

Table I ($\alpha = .05$)

Exact values for the .05 level of significance and the corresponding normal approximation for Max.$(m,n) \leq 20$ or $m + n \leq 30$.

m-n	n=1 E	1 N	2 E	2 N	3 E	3 N	4 E	4 N	5 E	5 N	6 E	6 N	7 E	7 N	8 E	8 N
0	-	-	-	-	0	1	1	2	4	5	7	8	11	12	15	16
1	-	-	-	-	0	1	2	3	5	6	8	9	13	14	18	19
2	-	-	-	-	1	2	3	4	6	7	10	11	15	16	20	21
3	-	-	0	1	2	3	4	5	8	9	12	13	17	18	23	24
4	-	-	0	1	2	3	5	6	9	10	14	15	19	20	26	27
5	-	-	0	1	3	4	6	7	11	12	16	17	21	23	28	29
6	-	-	1	2	4	5	7	8	12	13	17	18	24	25	31	32
7	-	-	1	2	4	5	8	9	13	14	19	20	26	27	33	35
8	-	-	1	2	5	6	9	10	15	16	21	22	28	29	36	37
9	-	-	1	3	5	7	10	11	16	17	23	24	30	31	39	40
10	-	-	2	3	6	7	11	13	18	19	25	26	33	34	41	42
11	-	-	2	3	7	8	12	14	19	20	26	28	35	36	44	45
12	-	-	2	4	7	9	14	15	20	22	28	29	37	38	47	48
13	-	-	3	4	8	9	15	16	22	23	30	31	39	40	49	50
14	-	-	3	4	9	10	16	17	23	24	32	33	41	42	52	53
15	-	-	3	5	9	11	17	18	25	26	34	35	44	45		
16	-	-	4	5	10	11	18	19	26	27	36	37	46	47		
17	-	-	4	5	11	12	19	20	28	29	37	38				
18	0	0	4	6	11	13	20	21	29	30	39	40				
19	0	0	5	6	12	13	21	22	30	32						
20	0	0	5	6	13	14	22	23	32	33						
21	0	0	5	7	13	15	23	24								
22	0	0	6	7	14	15	24	25								
23	0	0	6	7	15	16										
24	0	0	6	8	15	17										
25	0	0	7	8												
26	0	0	7	8												

Table I (α = .05)

m-n	9 E	9 N	10 E	10 N	11 E	11 N	12 E	12 N	13 E	13 N	14 E	14 N	15 E	15 N	16 E	16 N
0	21	22	27	28	34	35	42	44	51	52	61	62	72	73	83	84
1	24	25	31	32	38	39	47	48	56	57	66	67	77	78	89	90
2	27	28	34	35	42	43	51	52	61	62	71	72	83	84	95	96
3	30	31	37	38	46	47	55	56	65	66	77	78	88	90	101	102
4	33	34	41	42	50	51	60	61	70	71	82	83	94	95	107	108
5	36	37	44	45	54	55	64	65	75	76	87	88	100	101		
6	39	40	48	49	57	59	68	69	80	81	92	93				
7	42	43	51	52	61	62	72	73	84	85						
8	45	46	55	56	65	66	77	78								
9	48	49	58	59	69	70										
10	51	52	62	63												
11	54	55														
12	57	58														

m-n	17 E	17 N	18 E	18 N	19 E	19 N	20 E	20 N
0	96	97	109	110	123	124	138	139
1	102	103	116	117	130	131		
2	109	110	123	124				
3	115	116						

m-n	1 E	1 N	2 E	2 N	3 E	3 N	4 E	4 N
27	0	0						
28	0	0						
limit value for normal approximation as m → ∞	0.025		0.028		0.678		1.050	

γ Table II (α = .005)

δ - ~~.005~~ where t + 1 is the smallest integer such that
$\delta = P(U \leq t + 1 | H_0) > .005$ for $m + n \leq 30$ and Min.(m,n) $\leq$ 12.

m-n \ n	1	2	3	4	5	6	7	8
0	-	-	-	-	.003968	.003247	.002040	.001709
1	-	-	-	-	.004329	.002914	.002331	.001604
2	-	-	-	.004762	.002525	.002331	.001836	.001554
3	-	-	-	.003030	.002331	.002198	.001954	.001482
4	-	-	-	.004040	.002498	.001748	.001540	.001421
5	-	-	-	.002797	.002331	.001616	.001608	.001356
6	-	-	.004545	.002997	.002289	.001885	.001342	.001063
7	-	-	.003497	.002198	.002101	.001622	.001393	.001032
8	-	-	.002747	.002747	.002101	.001496	.001167	.000998
9	-	-	.004396	.002101	.001548	.001308	.001207	.000966
10	-	-	.003571	.001961	.001483	.001206	.001040	.000935
11	-	-	.002941	.002322	.001474	.001347	.001071	.000905
12	-	-	.003676	.001858	.001405	.001211	.000929	.000877
13	-	-	.003096	.001838	.001397	.001124	.000955	.000850
14	-	-	.002632	.001504	.001341	.001021	.000842	.000824
15	-	-	.002256	.001694	.001318	.000953	.000864	
16	-	-	.002597	.001694	.001277	.001038	.000909	
17	-	.004762	.002259	.001423	.001041	.000956		
18	-	.004329	.001976	.001538	.001028	.000896		
19	-	.003953	.002174	.001311	.001002			
20	-	.003623	.001923	.001319	.000989			
21	-	.003333	.001709	.001137				
22	-	.003077	.002137	.001241				
23	-	.002849	.001916					
24	-	.002646	.001724					
25	-	.004926						
26	-	.004598						

Table II ($\alpha = .005$)

m-n	n = 9	10	11	12
0	.001419	.001283	.001026	.000876
1	.001256	.001072	.000974	.000919
2	.001387	.001098	.000929	.000826
3	.001235	.001104	.000882	.000749
4	.001112	.000942	.000841	.000776
5	.001003	.000951	.000801	.000707
6	.000914	.000823	.000764	.000643
7	.000982	.000827	.000730	.000665
8	.000897	.000830	.000652	.000612
9	.000825	.000731	.000669	
10	.000759	.000732		
11	.000802			
12	.000742			

γ

Table II ($\alpha = .01$)

δ - .01 where $t + 1$ is the smallest integer such that
$\delta = P(U \leqq t + 1 | H_0) > .01$ for $m + n \leqq 30$ and Min.$(m,n) \leqq 12$.

m-n \ n	1	2	3	4	5	6	7	8
0	-	-	-	-	.007937	.005411	.004371	.002953
1	-	-	-	.007937	.006494	.004079	.003263	.002715
2	-	-	-	.009524	.006313	.004662	.003234	.002491
3	-	-	-	.006061	.005439	.003996	.003239	.002302
4	-	-	.008333	.006061	.004995	.003247	.002514	.002135
5	-	-	.006061	.006993	.004329	.002828	.002520	.002359
6	-	-	.009091	.004995	.004121	.003124	.002503	.002183
7	-	-	.006993	.004396	.003717	.002617	.002073	.002031
8	-	-	.005595	.004945	.003501	.002322	.002064	.001897
9	-	-	.006593	.003782	.003182	.002488	.002052	.001778
10	-	-	.005357	.003595	.003031	.002171	.002031	.001671
11	-	.009524	.004412	.003870	.002801	.002308	.001727	.001577
12	-	.008333	.004902	.003096	.002658	.002080	.001719	.001491
13	-	.007353	.004[illegible]28	.003008	.002496	.001858	.001706	.001413
14	-	.006536	.004386	.003144	.002376	.001950	.001692	.001343
15	-	.005848	.003759	.002597	.002240	.001784	.001480	
16	-	.005263	.003247	.002541	.002144	.001614	.001471	
17	-	.009524	.003953	.002688	.002031	.001593		
18	-	.008658	.003458	.002274	.001954	.001556		
19	-	.007905	.003478	.002222	.001861			
20	-	.007246	.003077	.002295	.001789			
21	-	.006667	.002735	.001979				
22	-	.006154	.003053	.001970				
23	-	.005698	.002737					
24	-	.005291	.002463					
25	-	.004926						
26	-	.004598						

Table II ($\alpha = .01$)

m-n	n = 9	10	11	12
0	.002838	.002344	.002038	.001609
1	.002403	.002245	.001853	.001610
2	.002072	.001840	.001699	.001413
3	.002164	.001794	.001561	.001411
4	.001890	.001735	.001444	.001404
5	.001942	.001684	.001512	.001253
6	.001717	.001628	.001401	.001246
7	.001761	.001396	.001304	.001122
8	.001572	.001362	.001218	.001117
9	.001605	.001325	.001141	
10	.001445	.001291		
11	.001311			
12	.001336			

Table II ($\alpha = .025$)

δ - ~~.025~~ γ where t + 1 is the smallest integer such that
$\delta = P(U \leq t + 1|H_0) > .025$ for $m + n \leq 30$ and Min.(m,n) $\leq$ 12.

m-n \ n	1	2	3	4	5	6	7	8
0	-	-	-	.014286	.011905	.011905	.008159	.007537
1	-	-	-	.015873	.010823	.008159	.006993	.006417
2	-	-	.017857	.014286	.012626	.008325	.006556	.005533
3	-	-	.023810	.015152	.010101	.008392	.006119	.004869
4	-	-	.016667	.012121	.008991	.006993	.005719	.005017
5	-	-	.018182	.008392	.007659	.006949	.005358	.004467
6	-	.022222	.013636	.008991	.006868	.005710	.005057	.004006
7	-	.018182	.013986	.008059	.007595	.005750	.004764	.004138
8	-	.015152	.010989	.008242	.006653	.004954	.004515	.003745
9	-	.012821	.010989	.007563	.006020	.004976	.003467	.003825
10	-	.021978	.008921	.007516	.005418	.004986	.004074	.003490
11	-	.019048	.010294	.006966	.004963	.004309	.003882	.003203
12	-	.016667	.008578	.007018	.005316	.004331	.003709	.003273
13	-	.014706	.008256	.005681	.004844	.003840	.003548	.003021
14	-	.013072	.007018	.005332	.004494	.003861	.003402	.002800
15	-	.011696	.007519	.005308	.004141	.003871	.003266	
16	-	.010526	.006494	.005082	.004393	.003451	.003140	
17	-	.009524	.006776	.005059	.004100	.003465		
18	-	.008658	.005928	.004816	.003816	.003136		
19	-	.011858	.006087	.004786	.003579			
20	-	.010870	.005385	.004103	.003761			
21	-	.010000	.005470	.003958				
22	-	.009231	.004884	.003941				
23	-	.008547	.005200					
24	-	.007937	.004680					
25	-	.007389						
26	-	.006897						

Table II (α = .025)

m-n	n = 9	10	11	12
0	.005162	.004584	.004180	.003481
1	.005001	.004190	.003670	.003306
2	.004870	.003877	.003651	.003145
3	.004705	.004064	.003244	.002996
4	.003989	.003754	.003227	.002859
5	.003893	.003493	.002902	.002732
6	.003790	.003257	.002887	.002616
7	.003691	.003053	.002864	.002508
8	.003228	.002867	.002606	.002408
9	.003161	.002961	.002588	
10	.003087	.002791		
11	.003017			
12	.002696			

γ

Table II ($\alpha = .05$)

δ - ~~.05~~ where t + 1 is the smallest integer such that
$\delta = P(U \leq t + 1 | H_0) > .05$ for m + n $\leq$ 30 and Min.(m,n) $\leq$ 12.

m-n \ n	1	2	3	4	5	6	7	8
0	-	-	.050000	.028571	.027778	.019481	.015443	.010956
1	-	-	.028571	.023810	.021645	.015486	.013364	.010613
2	-	-	.035714	.023810	.016414	.013320	.011713	.008981
3	-	.047619	.035714	.018182	.017094	.012787	.010490	.008904
4	-	.035714	.025000	.018182	.014486	.012238	.009427	.008693
5	-	.027778	.024242	.015385	.014985	.011635	.008573	.007573
6	-	.044444	.022727	.014985	.012592	.009696	.008914	.007480
7	-	.036364	.017483	.013158	.010989	.009325	.008153	.006612
8	-	.030303	.019231	.012637	.011321	.009004	.007511	.006518
9	-	.025641	.015385	.011345	.009890	.008698	.006955	.006434
10	-	.021978	.014286	.011111	.010126	.008391	.007183	.005788
11	-	.019048	.014706	.010062	.009091	.007930	.006686	.005712
12	-	.025000	.012255	.011146	.008126	.007014	.006242	.005641
13	-	.022059	.012384	.010693	.008291	.006832	.005855	.005138
14	-	.019608	.012281	.009843	.007552	.006654	.005507	.005077
15	-	.017544	.010526	.009486	.007698	.006480	.005644	
16	-	.015789	.010390	.008846	.007023	.006312	.005327	
17	-	.014286	.010728	.008538	.007135	.005652		
18	.050000	.012987	.009387	.008027	.006583	.005531		
19	.047619	.015810	.009130	.007749	.006071			
20	.045455	.014493	.009230	.007326	.006175			
21	.043478	.013333	.008205	.007115				
22	.041667	.012308	.008242	.006751				
23	.040000	.011396	.008210					
24	.038462	.010582	.007389					
25	.037037	.012315						
26	.035714	.011494						

Table II ($\alpha = .05$)

m-n	n = 9	10	11	12
0	.009790	.007956	.006735	.005898
1	.009017	.007924	.006433	.005953
2	.008383	.007056	.006153	.005494
3	.007801	.006359	.005891	.005095
4	.007312	.006351	.005648	.005119
5	.006864	.005768	.005420	.004773
6	.006475	.005752	.004825	.004467
7	.006119	.005277	.004660	.004195
8	.005804	.005259	.004505	.004212
9	.005523	.004855	.004358	
10	.005256	.004838		
11	.005017			
12	.004800			

m-n	n = 1
27	.034483
28	.033333

.005

Table III (α = .005)

$\delta - \gamma$ where t is the largest integer such that $\gamma = P(U \leqq t|H_0) \leqq .005$ and $\delta = P(U \leqq t + 1|H_0)$ for $m + n \leqq 30$ and $\text{Min.}(m,n) \leqq 12$.

m-n \ n	1	2	3	4	5	6	7	8
0	-	-	-	-	.001032	.000671	.001503	.001503
1	-	-	-	-	.000671	.000921	.000338	.001051
2	-	-	-	.000238	.002475	.001004	.001066	.000726
3	-	-	-	.001970	.001892	.001204	.000167	.000449
4	-	-	-	.000960	.001503	.001254	.000852	.000221
5	-	-	-	.002203	.001004	.001445	.000118	.000032
6	-	-	.000455	.001004	.000660	.000152	.000769	.000938
7	-	-	.001503	.002070	.000314	.000393	.000158	.000772
8	-	-	.002253	.001154	.000098	.000640	.000737	.000626
9	-	-	.000604	.002059	.001388	.000817	.000211	.000497
10	-	-	.001429	.001078	.001130	.001006	.000741	.000384
11	-	-	.002059	.000356	.000921	.000067	.000278	.000282
12	-	-	.000098	.001285	.000709	.000290	.000756	.000191
13	-	-	.000872	.000489	.000542	.000500	.000342	.000110
14	-	-	.001491	.001309	.000365	.000674	.000781	.000036
15	-	-	.001992	.000709	.000219	.000841	.000407	
16	-	-	.000455	.000012	.000074	.000105	.000041	
17	-	.000238	.001047	.000810	.000987	.000295		
18	-	.000671	.001542	.000251	.000849	.000471		
19	-	.001047	.000217	.000954	.000714			
20	-	.001377	.000769	.000409	.000593			
21	-	.001667	.001239	.001042				
22	-	.001923	.000116	.000584				
23	-	.002151	.000621					
24	-	.002354	.001059					
25	-	.000074						
26	-	.000402						

Table III (α = .005)

m-n	n = 9	10	11	12
0	.001113	.000535	.000825	.000853
1	.001190	.001025	.000687	.000222
2	.000154	.000522	.000576	.000443
3	.000356	.000056	.000489	.000645
4	.000545	.000574	.000423	.000146
5	.000718	.000192	.000372	.000371
6	.000872	.000657	.000334	.000573
7	.000179	.000330	.000306	.000163
8	.000376	.000022	.000285	.000375
9	.000547	.000463	.000271	
10	.000714	.000192		
11	.000156			
12	.000338			

Table III ($\alpha = .01$)

$\delta - \gamma$ where t is the largest integer such that $\gamma = P(U \leq t|H_0) \leq .01$ and $\delta = P(U \leq t + 1|H_0)$ for $m + n \leq 30$ and Min.(m,n) ≤ 12.

m-n \ n	1	2	3	4	5	6	7	8
0	-	-	-	-	.002063	.002424	.001259	.002618
1	-	-	-	.002063	.001342	.003007	.003007	.002390
2	-	-	-	.000471	.001162	.000010	.001783	.002230
3	-	-	-	.003939	.000676	.001209	.000744	.002128
4	-	-	.001667	.001919	.000509	.002008	.002301	.003070
5	-	-	.003939	.000210	.000343	.002728	.001466	.000054
6	-	-	.000909	.003007	.000385	.000896	.000699	.000174
7	-	-	.003007	.001208	.000304	.001633	.002071	.000294
8	-	-	.004405	.000110	.000313	.002312	.001422	.000412
9	-	-	.001209	.002437	.000282	.000823	.000818	.000525
10	-	-	.002857	.001176	.000325	.001503	.000257	.000634
11	-	.000476	.004118	.000196	.000319	.000143	.001460	.000738
12	-	.001667	.001422	.002157	.000355	.000862	.000964	.000837
13	-	.002647	.002776	.001245	.000371	.001462	.000494	.000931
14	-	.003464	.000351	.000294	.000401	.000305	.000050	.001021
15	-	.004152	.001729	.001982	.000420	.000926	.001111	
16	-	.004737	.002857	.001154	.000453	.001458	.000705	
17	-	.000476	.000966	.000435	.000474	.000453		
18	-	.001342	.002095	.001906	.000507	.000997		
19	-	.002095	.000000	.001168	.000527			
20	-	.002754	.001154	.000525	.000555			
21	-	.003333	.002137	.001832				
22	-	.003846	.000537	.001206				
23	-	.004302	.001516					
24	-	.004709	.002365					
25	-	.000148						
26	-	.000805						

Table III (α = .01)

m-n	n = 9	10	11	12
0	.000621	.000728	.000385	.001364
1	.001394	.000168	.000603	.000608
2	.002052	.001550	.000821	.001333
3	.000807	.001112	.001032	.000705
4	.001506	.000727	.001235	.000126
5	.000434	.000394	.000087	.000844
6	.001130	.000100	.000359	.000344
7	.000196	.001238	.000612	.000998
8	.000869	.000972	.000845	.000557
9	.000024	.000731	.001061	
10	.000682	.000513		
11	.001245			
12	.000546			

$\delta - \gamma$ where t is the largest integer such that $\gamma = P(U \leq t|H_0) \leq .025$ and $\delta = P(U \leq t + 1|H_0)$ for m + n ≤ 30 and Min.(m,n) ≤ 12.

m-n \ n	1	2	3	4	5	6	7	8
0	-	-	-	.010714	.009127	.004437	.006061	.000058
1	-	-	-	.009127	.009848	.007517	.004953	.001800
2	-	-	.007143	.005952	.001010	.003688	.004108	.003290
3	-	-	.001190	.003788	.002467	.000225	.003455	.004559
4	-	-	.008333	.000758	.004021	.004021	.002941	.001328
5	-	-	.000758	.008217	.005020	.001164	.002534	.002724
6	-	.002778	.006818	.007018	.005998	.004261	.002219	.003925
7	-	.006818	.000524	.005220	.000760	.001891	.001952	.001348
8	-	.009848	.005769	.004121	.002007	.004618	.001739	.002589
9	-	.012179	.000824	.002731	.003152	.002573	.001562	.000263
10	-	.003022	.005357	.001797	.004102	.000636	.001415	.001506
11	-	.005952	.001471	.000748	.004950	.003117	.001291	.002604
12	-	.008333	.005392	.000026	.001153	.001404	.001189	.000626
13	-	.010294	.001264	.004783	.002146	.003611	.001101	.001715
14	-	.011928	.004825	.003811	.003049	.002071	.001027	.002689
15	-	.001608	.001692	.003091	.003826	.000592	.000964	
16	-	.003947	.004870	.002320	.000676	.002621	.000910	
17	-	.005952	.001849	.001680	.001576	.001273		
18	-	.007684	.004743	.000987	.002371	.003106		
19	-	.001285	.001957	.000442	.003098			
20	-	.003261	.004615	.003950	.000398			
21	-	.005000	.002094	.003317				
22	-	.006539	.004548	.002778				
23	-	.007906	.002285					
24	-	.001190	.004557					
25	-	.002833						
26	-	.004310						

Table III (α = .025)

m-n	n = 9	10	11	12
0	.005008	.003371	.001346	.002549
1	.003263	.003518	.003070	.002144
2	.001774	.003715	.001276	.001823
3	.000464	.000335	.002886	.001567
4	.003312	.000798	.001369	.001362
5	.002208	.001238	.002864	.001198
6	.001208	.001646	.001542	.001067
7	.000308	.002031	.000304	.000963
8	.002717	.002390	.001749	.000880
9	.001868	.000022	.000641	
10	.001150	.000484		
11	.000455			
12	.002507			

Table III ($\alpha = .05$)

$\delta - \gamma$ where t is the largest integer such that $\gamma = P(U \leq t|H_0) \leq .05$ and $\delta = P(U \leq t + 1|H_0)$ for $m + n \leq 30$ and $Min.(m,n) \leq 12$.

m-n	n = 1	2	3	4	5	6	7	8
0	-	-	.000000	.021429	.002381	.003463	.001340	.008508
1	-	-	.021429	.018254	.008874	.013288	.003069	.003640
2	-	-	.014286	.016667	.013384	.009374	.004633	.008430
3	-	.002381	.002381	.013636	.003380	.006044	.006088	.004579
4	-	.014286	.016667	.013636	.008541	.003297	.007328	.001060
5	-	.022222	.007576	.012238	.000383	.000873	.008442	.005477
6	-	.005556	.018182	.012038	.004899	.008468	.001600	.002491
7	-	.013636	.011538	.011172	.009114	.006287	.003113	.006337
8	-	.019697	.006044	.010989	.002731	.004334	.004458	.003699
9	-	.024359	.014835	.010504	.006484	.000870	.005653	.001254
10	-	.006044	.008929	.010458	.000851	.000987	.000255	.004752
11	-	.011905	.004412	.010010	.004494	.005313	.001650	.002543
12	-	.000000	.012010	.000258	.007545	.005236	.002901	.000469
13	-	.005882	.007688	.000710	.002837	.003840	.004034	.003649
14	-	.010784	.003509	.000923	.005816	.002547	.005062	.001744
15	-	.014912	.010150	.001327	.001515	.001346	.000805	
16	-	.002632	.006494	.001534	.004348	.000226	.001944	
17	-	.007143	.003134	.001858	.000390	.004836		
18	.000000	.011039	.008992	.002040	.003114	.003737		
19	.002381	.002569	.005652	.002308	.005522			
20	.004545	.006522	.002692	.002479	.002037			
21	.006522	.010000	.007949	.002718				
22	.008333	.000769	.005128	.002855				
23	.010000	.004416	.002381					
24	.011538	.007672	.007143					
25	.012963	.000739						
26	.014286	.004023						

Table III ($\alpha = .05$)

m-n	n = 9	10	11	12
0	.003044	.005395	.006027	.005633
1	.002640	.000691	.029159	.001179
2	.002411	.003455	.003346	.002494
3	.002274	.005853	.002287	.003695
4	.002221	.002179	.001369	.000049
5	.002213	.004514	.000567	.001365
6	.002246	.001293	.004689	.002567
7	.002301	.003539	.003901	.003668
8	.002377	.000668	.003193	.000728
9	.002396	.002804	.002552	
10	.002559	.000212		
11	.002659			
12	.002763			

m-n	n = 1
27	.015517
28	.016667

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